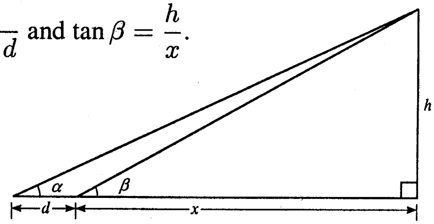
48. (a) Solution #1: Let x be the base of the smaller triangle, as shown in the figure on the right. We see that $x + d = h \cdot \cot\alpha$ and $x = h \cdot \cot\beta$. Therefore, $h \cdot \cot\alpha - d = h \cdot \cot\beta \quad \Leftrightarrow$ $h \cdot \cot\alpha - h \cdot \cot\beta = d \quad \Leftrightarrow \quad h(\cot\alpha - \cot\beta) = d \quad \Leftrightarrow$

$$h = \frac{d}{\cot\alpha - \cot\beta} = \frac{d}{\frac{1}{\tan\alpha} - \frac{1}{\tan\beta}} = \frac{d}{\frac{\tan\beta - \tan\alpha}{\tan\alpha \cdot \tan\beta}} = d \cdot \frac{\tan\alpha \cdot \tan\beta}{\tan\beta - \tan\alpha}$$

Solution #2: From the figure, we see that $\tan\alpha = \frac{h}{x+d}$ and $\tan\beta = \frac{h}{x}$.



Solving each of these equations for x, we find that

$x = \frac{h}{\tan\beta}$ and $x = \frac{h}{\tan\alpha} - d$, so $\frac{h}{\tan\beta} = \frac{h}{\tan\alpha} - d$

$$\Rightarrow \quad \frac{h}{\tan\beta} - \frac{h}{\tan\alpha} = -d \quad \Rightarrow$$

$$h\left(\frac{1}{\tan\alpha} - \frac{1}{\tan\beta}\right) = d \quad \Leftrightarrow \quad h\left(\frac{\tan\beta - \tan\alpha}{\tan\alpha \cdot \tan\beta}\right) = d \quad \Leftrightarrow \quad h = d \cdot \frac{\tan\alpha \cdot \tan\beta}{\tan\beta - \tan\alpha}$$

To complete this part of the exercise we show that

$$h = d \cdot \frac{\tan\alpha \cdot \tan\beta}{\tan\beta - \tan\alpha} = d \cdot \frac{\tan\alpha \cdot \tan\beta}{\tan\beta - \tan\alpha} \cdot \frac{\frac{1}{\tan\alpha \cdot \tan\beta}}{\frac{1}{\tan\alpha \cdot \tan\beta}} = \frac{d}{\frac{1}{\tan\alpha} - \frac{1}{\tan\beta}} = \frac{d}{\cot\alpha - \cot\beta}$$

(b) $h = d \cdot \frac{\tan\alpha \cdot \tan\beta}{\tan\beta - \tan\alpha} = (800) \cdot \frac{\tan 25^\circ \cdot \tan 29^\circ}{\tan 29^\circ - \tan 25^\circ} \approx 2350$ ft.

50. (a) $s = r\theta \quad \Leftrightarrow \quad \theta = \frac{s}{r} = \frac{6155}{3960} \approx 1.5543 \text{ rad} \approx 89.05^\circ$

(b) Let d represent the distance, in miles, from the center of the earth to the moon. Since $\cos\theta = \frac{3960}{d}$, we have $d = \frac{3960}{\cos\theta} \approx \frac{3960}{\cos 89.05^\circ} \approx 239{,}961.5$. So the distance AC is $239{,}961.5 - 3960 \approx 236{,}000$ mi.

52. Let d represent the distance, in miles, from the earth to Alpha Centauri. Since $\sin 0.000211^\circ = \frac{93{,}000{,}000}{d}$, we have $d = \frac{93{,}000{,}000}{\sin 0.000211^\circ} \approx 25{,}253{,}590{,}022{,}410$. So the distance from the earth to Alpha Centauri is about 2.53×10^{13} mi.

54. Let d be the length of the base of the 60° triangle. Then $\tan 60^\circ = \frac{85}{d} \quad \Leftrightarrow$ $d = \frac{85}{\tan 60^\circ} \approx 49.075$, and so $\tan 30^\circ = \frac{85}{d+x} \quad \Leftrightarrow \quad d + x = \frac{85}{\tan 30^\circ} \quad \Leftrightarrow$ $x = \frac{85}{\tan 30^\circ} - d \approx 98.1$.

56. Let h be the hypotenuse of the top triangle. Then $\sin 30^\circ = \frac{5}{h} \quad \Leftrightarrow \quad h = \frac{5}{\sin 30^\circ} = 10$, and so $\tan 60^\circ = \frac{h}{x} \quad \Leftrightarrow \quad x = \frac{h}{\tan 60^\circ} = \frac{10}{\tan 60^\circ} \approx 5.8$.